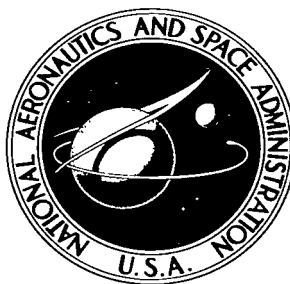


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DESIGN EQUATIONS OF SWEPT FREQUENCY SPECTRUM ANALYZERS FOR IN-FLIGHT VIBRATION ANALYSIS

by Edward F. Miller and David L. Wright

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SUMMARY

The equations for the design of swept frequency spectrum analyzers are presented with the intended application being in-flight analysis of vibration data. Primary emphasis is given to developing the optimum combination of the frequency-time function and the analyzing filter bandwidth. The main criterion used is minimum sweep time coupled with acceptable resolution and accuracy. Constant, linear, and second-order sweep rates are discussed, and the frequency-time equation is developed for each of these three types. The analyzer using a second-order sweep rate is shown to have the shortest sweep time, but it requires an expanding bandwidth filter and a decreasing averaging-time power detector. The linear sweep rate analyzer is shown to require less bandwidth per channel and is much less complex. The constant sweep rate analyzer is shown to have either very long sweep times or poor frequency resolution; however, all three types of analyzers have bandwidth savings over real-time transmission of wide-band data.

INTRODUCTION

A continuing problem in communication system design is transmitting many channels of wide-band data over a limited bandwidth transmission system. The problem is most critical in transmission from a spacecraft or a launch vehicle, where increased bandwidth requires more weight and power, and consequently, reduced payload. The particular problem that led to this investigation was the need to transmit from launch vehicles the results of a large number of vibration measurements.

Currently, NASA uses primarily the IRIG FM/FM (Inter-Range Instrumentation Group) standard telemetry format for the transmission of analog vibration signals from vehicles to the ground, and an extensive network of ground stations has been built to receive data in this format. Often, the number of vibration measurements desired exceeds

the capacity of the IRIG FM/FM telemetry system. There are several ways to solve this problem. One method is to change the transmission system. Two transmission systems capable of transmitting many channels of real-time, wide-band data from space vehicles are DSB/FM (double-sideband/frequency modulation (ref. 1)) and SSB/FM (single-sideband/frequency modulation (ref. 2)). Another alternative is to use some form of data reduction before the data is transmitted. On-board data reduction reduces the transmission bandwidth required and still allows the use of the IRIG FM/FM transmission system for the resulting low bandwidth channels. Spectrum analysis is the method of pretransmission data reduction that is considered in this report.

Spectrum analysis of a signal often gives sufficient information about the real-time signal. In many cases, the final form of vibration data is a plot of the power density of the signal as a function of frequency (or power spectral density). This form of the result is important because it can show the frequencies present in the vibration source as well as the amplitudes and frequencies of the dominant vibration modes of the vehicle structure. However, the process of spectrum analysis is irreversible since no phase relations between the frequency components of a given signal are obtained; thus, spectrum analysis loses the actual time history of the vibration. In addition, relative phase measurements between two time functions cannot be made by comparing their spectrums. Even with these limitations spectrum analysis remains useful.

Theoretical work in spectrum analysis (refs. 3 and 4) was followed by the consideration of swept frequency spectrum analyzers with a constant sweep rate and a constant bandwidth filter (refs. 5 and 6). Commercially available airborne spectrum analyzers are of this type; however, they have the drawbacks of poor resolution and/or long sweep times. For random data, a spectrum analyzer with a linear sweep rate has better resolution and/or shorter sweep time than the constant sweep rate analyzer. The linear sweep rate analyzers have been used (to the authors' knowledge) only as ground equipment. A spectrum analyzer with a sweep rate proportional to frequency squared was mentioned by Bendat (ref. 7), but the frequency-time function was not derived.

In this report, the basic theoretical background for spectrum analysis is presented. The frequency-time functions are derived for the various types of swept frequency spectrum analyzers. The development progresses from the simple case of a constant sweep rate analyzer operating on periodic signals through to the progressively more complex operations of constant, linear, and second-order sweep rate systems analyzing random signals. For each type of frequency sweep, the attainable performance, as given by the frequency-time function, is compared with the following desired specifications for airborne spectrum analyzers (the list was compiled from discussions with those who use and reduce vibration data):

Frequency range	near zero to several kilohertz
Allowable frequency error	± 10 percent of the frequency being analyzed
Minimum resolvable bandwidth	± 10 percent of the frequency being analyzed
Allowable spectrum amplitude error	± 10 percent
Sweep time	of the order of seconds

These desired specifications are shown to be incompatible and cannot be obtained in the analysis of random data. Even periodic data can only be analyzed with less than the desired accuracy in such a short sweep time.

In those cases where long sweep times or reduced accuracies are tolerable, the in-flight spectrum analyzer may be used. The bandwidth savings resulting from the use of spectrum analyzers in these cases is discussed in the concluding section of this report.

A limitation of the swept frequency spectrum analyzers discussed in this report is that it is not practical to extend their lower frequency limit to "near zero," as listed in the desired specifications previously given. The frequency-time equations derived in this report show that low-frequency spectrum analyzers have extremely long sweep times and require very narrow filters to obtain good frequency resolution. The swept frequency spectrum analyzers have a practical low-frequency limit on the order of 100 hertz. The low-frequency part of the signal is better handled by separate transmission of that portion of the time-varying signal followed by spectrum analysis on the ground.

BACKGROUND TO PROBLEM

Before proceeding to the details of the analysis, some necessary background information will be discussed.

General Background

Characteristics of expected signals. - Since this report is concerned with the specific problem of transmitting vibration data, the assumed characteristics of such data will be discussed. First, the source causing the vibration is assumed to be a random noise source. The mechanical system coupled to this source acts as a filter that shapes the frequency content of the resulting vibration. The mechanical Q , or sharpness of resonance, is assumed to be less than 10; thus, if the spectral peaks are to be resolved, analyzing filters with values of electrical Q much greater than 10 would be required. Moreover, the maximum mechanical Q is assumed to be independent of the resonant frequency. Thus, the bandwidths of the resonances become progressively wider as the frequency increases.

In order to obtain meaningful data, the signal being analyzed must be stationary during the time of observation; that is, the probability distribution of the signal level must remain constant. The vibration signal will remain stationary as long as the source causing the vibration remains statistically invariant. Examples of events causing nonstationarity would be turning on or turning off a rocket engine, changes in thrust, or changes in air turbulence surrounding the vehicle.

For the swept frequency spectrum analyzer, the minimum period of stationarity of the signal limits the total sweep time. When the signal is nonstationary, the spectrum changes with time. Thus, for the swept frequency spectrum analyzer the signal must be stationary for times on the order of the total sweep time. The minimum period of stationarity can be somewhat less than the total sweep time since interpolation between succeeding spectrums can give estimates of the spectrum between sweeps.

Types of spectrum analyzers. - There are two common types of analog spectrum analyzers. The simplest method of spectrum analysis is to measure the power from each filter in a group of band-pass filters whose center frequencies are spaced so that the filter pass bands cover the frequency range of the input signal. The power output of each filter is an approximation to the average power spectral density of the signal within the filter pass band. The swept frequency spectrum analyzer is the second common type of spectrum analyzer and is the subject of this report. Its operation will be discussed in detail.

The analyzer shown in figure 1 is one of several ways of implementing the swept frequency analyzer. The frequency diagram is shown in figure 2. Basically, the analyzer in figure 1 translates the signal spectrum to a higher frequency using suppressed carrier amplitude modulation. At the high frequency, a single band-pass filter selects a narrow band of frequencies corresponding to a similar narrow band of frequencies of the input

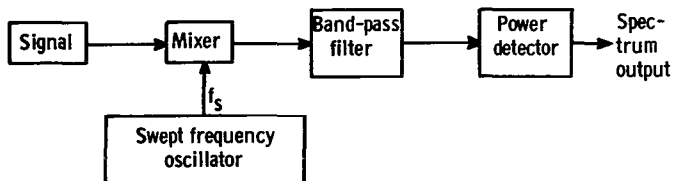


Figure 1. - Block diagram of typical swept frequency spectrum analyzer.

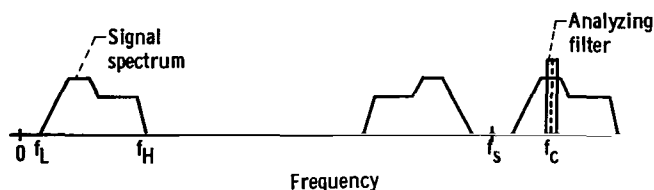


Figure 2. - Frequency diagram of typical swept frequency spectrum analyzer. Lowest frequency to be analyzed f_L ; highest frequency to be analyzed f_H ; mixing frequency f_s ; center frequency f_c .

spectrum. A sample of the power output of the filter is an approximation to the power spectral density within the corresponding signal frequency increment. The filter is effectively swept through the signal spectrum by sweeping the mixing frequency f_s . A disadvantage of this analyzer is that both the filter and the power detector must have time to change their output values as they are swept through the spectrum. A major objective of this report is to show that the proper choice of the time dependence of the frequency sweep will

optimize the sweep time and the resolution for the swept frequency spectrum analyzer. Before deriving the frequency-time equations for the constant, the linear, and the second-order sweep rate spectrum analyzers, some of the basic theory of spectrum analysis will be presented.

Theoretical Background

The basic restrictions on swept frequency spectrum analysis will be shown by first considering the spectrum analysis of periodic signals. Then the more complex case of analysis of random signals will be considered.

Spectrum analysis of periodic signals. - Periodic signals are representable by a Fourier series of harmonically related sinusoidal waves. The basis for such representation is well known and will not be discussed here (see ref. 8 for a discussion of Fourier series).

The spectrum for a periodic signal has discrete components whose amplitudes are constant with time. Such a spectrum may be resolved by a swept frequency spectrum analyzer with a sharp band-pass filter whose bandwidth is less than the minimum spacing between frequency components of the spectrum. Thus,

$$B_f < \frac{1}{T_p} \quad (1)$$

where

B_f effective filter bandwidth (bandwidth of the ideal rectangular filter that passes the same power as the real filter when both are excited by white noise)

T_p period of the signal

Furthermore, the sweep rate of the analyzer must be slow enough to allow the filter response to come up to almost full value before the analyzer moves on to another frequency. A band-pass filter excited by a sinusoidal signal at its center frequency has an output response envelope with a time constant given by

$$T_c \approx \frac{1}{\pi B_f} \quad (2)$$

where T_c is the time constant of response.

Also, the output of the filter must be measured by a detector whose averaging time is long compared with the period of the filter output, which is a sine wave at the filter center frequency. Thus, the averaging time is given by

$$T_d \gg \frac{1}{f_c} \quad (3)$$

where

T_d detector averaging time

f_c filter center frequency

Using equations (1), (2), and (3), one can design a spectrum analyzer for periodic signals.

Spectrum analysis of random signals. - The measurement of the spectrum of a random signal is not as straightforward. By its nature, a random signal has no periodicity, its value as a function of time cannot be precisely predicted, and, at best, only a probability distribution for its values can be specified. Analyzing such functions requires some of the techniques of generalized harmonic analysis and probability theory. A brief introduction to the required techniques follows (for a thorough discussion see refs. 9 to 11.).

The power spectral density (or mean square spectral density) of a stationary random function of time is defined as the Fourier transform of the autocorrelation function $R(\tau)$. Let $x(t)$ be the random function. Then

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) x(t+\tau) dt \quad (4)$$

and the power spectral density $S(f)$ is given by

$$S(f) = \int_{-\infty}^{\infty} R(\tau) e^{-j2\pi f\tau} d\tau \quad (5)$$

(Symbols are defined in the appendix.) The function $S(f)$ has the property that

$$\int_{-\infty}^{\infty} S(f) df = \text{Mean square value of } x(t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x^2(t) dt \quad (6)$$

Also,

$$2 \int_{f_a}^{f_b} S(f) df = P_{ab} \quad (7)$$

where P_{ab} is the average power in $x(t)$ contained between frequencies f_a and f_b , and between frequencies $-f_a$ and $-f_b$, where $f_b > f_a \geq 0$. If $S(f)$ is nearly constant between f_a and f_b , for example $S_{ab}(f)$, then $S_{ab}(f)$ may be approximated by

$$\hat{S}_{ab}(f) = \frac{P_{ab}}{2(f_b - f_a)} \quad (8)$$

where the caret above the S indicates that this is an estimated value.

The spectral density previously defined is two sided; that is, $S(f)$ is nonzero for both positive and negative frequencies. In engineering applications, spectral densities are usually considered one sided (nonzero only for positive frequencies). Hereinafter, the spectral densities will be considered one sided. Equation (8) becomes

$$\hat{S}_{ab}(f) = \frac{P_{ab}}{f_b - f_a} \quad (9)$$

This equation is the basis of the most common method of spectral analysis, the use of a narrow band-pass filter to measure the power in $x(t)$ between the frequencies f_a and f_b . If the effective filter bandwidth is B_f , then

$$\hat{S}_{ab}(f) \approx \frac{\bar{P}_{ab}}{B_f} \quad (10)$$

where \bar{P}_{ab} is the average power out of the filter whose effective bandwidth is B_f ; that is,

$$\bar{P}_{ab} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x_f^2(t) dt \quad (11)$$

where $x_f(t)$ is the output of the filter.

Rewriting equation (10), incorporating equation (11), and considering the limiting case yield the following expression for the exact spectral density:

$$S(f_c) = \lim_{B_f \rightarrow 0} \lim_{T \rightarrow \infty} \frac{1}{B_f T} \int_0^T x_f^2(t) dt \quad (12)$$

where

f_c center frequency of the filter

$S(f_c)$ exact value of the power spectral density at frequency f_c

For practical reasons, B_f cannot go to zero; however, a small value of B_f will give a close approximation as long as T is sufficiently large.

The estimated value of $S(f_c)$ can be written as

$$\hat{S}(f_c) = \frac{1}{B_f T} \int_0^T x_f^2(t) dt \quad (13)$$

In words, $\hat{S}(f_c)$ is the estimate of the average power spectral density in a bandwidth B_f centered about f_c . $\hat{S}(f_c)$ is an unbiased estimator of the spectral density at f_c only if the spectrum is constant in the bandwidth B_f . (However, for nonflat spectrums, $\hat{S}(f_c)$ is an unbiased estimator of the average spectral density within the bandwidth B_f .) Since $\hat{S}(f_c)$ is only an estimate of the true value of $S(f_c)$, the measured values of $\hat{S}(f_c)$ will have statistical variation about their average value.

When equation (13) is implemented, T is the averaging time of the power detector on the output of the filter. If the power detector implements equation (11) for finite T through an integrate and reset mechanism, true averaging is performed and T is the time between integrator resets. However, if a resistance-capacitance averaging circuit is used, T is two times the RC time constant (ref. 7, section 4).

The probability distribution of $\hat{S}(f_c)$ will indicate the expected range of values of these estimates of the spectral density. To determine the probability distribution of $\hat{S}(f_c)$ consider the following:

(1) From sampling theory, it can be established that the maximum number of independent samples obtainable from a band-limited white noise signal of bandwidth B_f and duration T is $n = 2B_f T$ (ref. 12).

(2) The variance estimate of a normal distribution with zero mean value is a function of the chi-square random variable with $N-1$ degrees of freedom, where N is the number of samples taken to determine the sample variance (ref. 13). Specifically,

$$\frac{s^2}{\sigma^2} (N-1) = \chi_{N-1}^2 \quad (14)$$

where

s^2 sample variance

σ^2 true variance

χ_{N-1}^2 chi-square random variable with $N-1$ degrees of freedom (the chi-square distribution is tabulated in statistical tables)

(3) For certain types of random functions, those which satisfy the quasi-ergodic hypothesis (ref. 11, ch. 7), the time average of the function equals the ensemble average; that is,

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x^2(t) dt = \int_{-\infty}^{\infty} p[x(t)] x^2(t) dx \quad (15)$$

where $p[x(t)]$ is the probability distribution of $x(t)$. Equation (15) states that the mean square value of $x(t)$ equals the second moment of $x(t)$. The random functions considered will be assumed to satisfy the quasi-ergodic hypothesis.

(4) For a random variable $x(t)$,

$$\underbrace{E[x^2(t)]}_{\substack{\text{Second} \\ \text{moment} \\ \text{of } x(t)}} = \underbrace{\text{Var } x(t)}_{\substack{\text{Variance} \\ \text{of } x(t)}} + \underbrace{\{E[x(t)]\}^2}_{\substack{\text{Mean value} \\ \text{of } x(t), \text{ quan-} \\ \text{tity squared}}} \quad (16)$$

Applying equation (6) and steps (1), (2), (3), and (4) leads to the following conclusion: The power measured at the output of an ideal band-pass filter fed by white noise is an estimate of the variance of the band-limited white noise (because the noise has zero mean value), and thus the power is a function of chi-square with $n-1$ or $2B_f T-1$ degrees of freedom. The filter power is assumed to be measured using an averaging time T .

Therefore $\hat{S}(f_c)$ is given by

$$\hat{S}(f_c) = S(f_c) \frac{\chi_{n-1}^2}{n-1} \quad (17)$$

The probability of $\hat{S}(f_c)$ being within a certain interval of $S(f_c)$ can be calculated as follows:

Let

$$P[S_1 \leq \hat{S}(f_c) \leq S_2] = 1-\alpha \quad (18)$$

where

$P[a \leq y \leq b]$ probability of $a \leq y \leq b$

$1-\alpha$ desired confidence level

S_1, S_2 to be determined

From equations (17) and (18)

$$\begin{aligned} P\left[S_1 \leq S(f_c) \frac{\chi_{n-1}^2}{n-1} \leq S_2\right] &= 1-\alpha \\ P\left[\frac{(n-1)S_1}{S(f_c)} \leq \chi_{n-1}^2 \leq \frac{(n-1)S_2}{S(f_c)}\right] &= 1-\alpha \end{aligned} \quad (19)$$

Let

$$P[X_1 \leq \chi_{n-1}^2 \leq X_2] = 1-\alpha \quad (20)$$

where X_1 is the point of the χ_{n-1}^2 distribution that has $\alpha/2$ area to the left and X_2 is the point that has $\alpha/2$ area to the right (X_1 and X_2 can be found in tables of the chi-square distribution). Then, from equations (19) and (20)

$$\left. \begin{aligned} S_1 &= \frac{X_1 S(f_c)}{n-1} \\ S_2 &= \frac{X_2 S(f_c)}{n-1} \end{aligned} \right\} \quad (21)$$

If we now define the normalized power spectral density estimate as $\hat{S}(f_c)/S(f_c)$, the following equation results:

$$P\left[\frac{X_1}{n-1} \leq \frac{\hat{S}(f_c)}{S(f_c)} \leq \frac{X_2}{n-1}\right] = 1-\alpha \quad (22)$$

This equation, together with a table of the chi-square probability distribution, can be used to determine the normalized power spectral density at the tabulated confidence levels.

As an example consider $\alpha = 20$ percent

$$B_f = 10 \text{ Hz}$$

$$T = 1.55 \text{ sec}$$

Then, the confidence level $1-\alpha$ is 80 percent, and

$$n-1 = 2B_f T - 1 = 31 - 1 = 30$$

The values from $\chi^2_{(30)}$ tables with $\alpha = 20$ percent are

$$X_1 = 20.60$$

$$X_2 = 40.26$$

Thus

$$P \left[0.686 \leq \frac{\hat{S}(f_c)}{S(f_c)} \leq 1.34 \right] = 80 \text{ percent}$$

For this numerical example, the probability of the measured spectral density lying in the interval $[0.686 S(f_c), 1.34 S(f_c)]$ is 80 percent.

It is unwieldy to use equation (22) to estimate the interval into which $\hat{S}(f_c)/S(f_c)$ falls $1-\alpha$ percent of the time. A much simpler expression is obtained by using the normal approximation to the chi-square distribution. For $n-1 \geq 30$, the chi-square distribution is approximated very closely by the normal distribution with mean μ and standard deviation σ :

$$\left. \begin{aligned} \mu &= n-1 \\ \sigma &= \sqrt{2(n-1)} \end{aligned} \right\} \quad (23)$$

Let

$$\frac{\hat{S}(f_c)}{S(f_c)} (n-1) = \chi^2 = w \quad (24)$$

where

w is the normal random variable, with mean μ and standard deviation σ .

One can find in normal probability tables the values of η that satisfy

$$P[-\eta \leq z \leq \eta] = 1-\alpha \quad (25)$$

where z is the standardized normal random variable ($\mu = 0$ and $\sigma = 1$). Now,

$$\frac{w - \mu}{\sigma} = z$$

and equation (25) becomes

$$P\left[-\eta \leq \frac{w - \mu}{\sigma} \leq \eta\right] = 1-\alpha \quad (26)$$

Using equations (23) and (24) in equation (26) results in

$$P\left[1-\eta\sqrt{\frac{2}{n-1}} \leq \frac{\hat{S}(f_c)}{S(f_c)} \leq 1+\eta\sqrt{\frac{2}{n-1}}\right] = 1-\alpha$$

or

$$P\left[1 - \epsilon \leq \frac{\hat{S}(f_c)}{S(f_c)} \leq 1 + \epsilon\right] = 1-\alpha \quad (27)$$

where the percent error ϵ is

$$\epsilon = \eta\sqrt{\frac{2}{n-1}} = \eta\sqrt{\frac{2}{2B_f T - 1}} \quad (28)$$

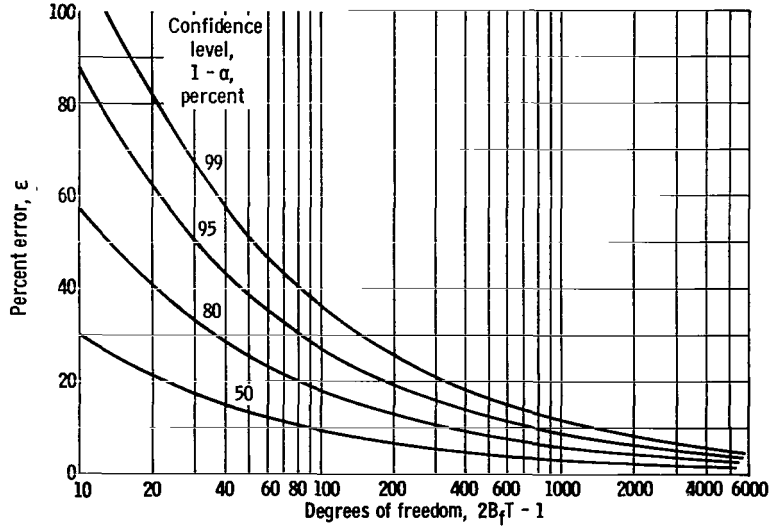


Figure 3. - Percent error as a function of the degrees of freedom for various confidence levels using the normal approximation to chi-square distribution (eq. (27)).

For $B_f T \gg 1$

$$\epsilon \approx \frac{\eta}{\sqrt{B_f T}} \quad (29)$$

Equation (27) is to be interpreted to mean that the normalized values of the estimated spectral density will lie in the interval $[(1 - \epsilon), (1 + \epsilon)]$ approximately $1 - \alpha$ percent of the time, with ϵ as given by equation (28).

Equation (27) is plotted in figure 3. In the region to the left of $(2B_f T - 1) = 30$, the graph gives only an approximation to the true confidence interval and is meant to be used as a general guide. For $(2B_f T - 1) = 30$, the interval limits are accurate to 1.1 percent for $\alpha = 0.50$ and 11.8 percent for $\alpha = 0.05$. If the normal approximation to the chi-square distribution is used in the previous numerical example, the following values are obtained:

$$\eta = 1.28$$

and

$$P \left[0.670 \leq \frac{\hat{S}(f_c)}{S(f_c)} \leq 1.33 \right] = 80 \text{ percent}$$

which agrees with the limits obtained previously within about 2.3 percent.

In summary, equation (22) is the exact expression used to obtain a confidence interval on the estimated spectral density when a confidence level is given. For large values of $B_f T$, equations (27) and (28) may be used.

The reader may recall equation (2) and might wonder why the response time of the analyzing filter did not enter into the analysis for random signals. The answer lies in the requirement that a number of independent samples of the power level be taken in each frequency interval. If the samples are to be truly independent, the filter output must have had ample time to change from one level to another. Thus, taking $2B_f T - 1$ independent samples in each frequency interval guarantees that the filter response time has been exceeded many times.

Now that sufficient background information has been introduced the design of spectrum analyzers will be discussed.

DESIGN

The theory of swept frequency spectrum analyzers will be applied first to the design for periodic signals. Then the theory will be applied to random signals, which are representative of flight vibration data.

Design for Periodic Signals

For periodic signals, there is no statistical uncertainty in the time function and equations (1), (2), and (3) apply:

$$B_f < \frac{1}{T_p} \quad (1)$$

$$T_c \approx \frac{1}{\pi B_f} \quad (2)$$

$$T_d \gg \frac{1}{f_c} \quad (3)$$

Equation (1) is satisfied by choosing a narrow band-pass filter as the analyzing filter. Equation (2) is satisfied by requiring each Fourier component to remain in the filter

long enough to allow the filter to come up to nearly full response. Let the sweep rate of the analyzer be such that in time $N_1 T_c$ the filter moves across a frequency span of width B_f (N_1 is a numerical constant). If $N_1 = 3$, the filter will be at approximately 95 percent of its full value as the Fourier component leaves the filter. For a constant sweep rate,

$$\frac{df_i}{dt} = K_1 \quad (30)$$

where

f_i signal frequency being analyzed

$\frac{df_i}{dt}$ sweep rate

K_1 sweep rate constant

Sweeping at a rate slow enough to allow adequate filter response and yet having minimum sweep time requires that

$$N_1 T_c = \frac{B_f}{\frac{df_i}{dt}} = \frac{B_f}{K_1} \quad (31)$$

Thus,

$$K_1 = \frac{B_f}{N_1 T_c} = \frac{\pi B_f^2}{N_1} \quad (32)$$

Solving equation (30) subject to $f_i(0) = f_L$, where f_L is the lowest frequency to be analyzed, yields

$$f_i(t) = K_1 t + f_L \quad (33)$$

The total sweep time t_s may be found by using $f_i(t_s) = f_H$, where f_H is the highest frequency to be analyzed, in equation (33). Then solving for t_s gives

$$t_s = \frac{f_H - f_L}{K_1} = \frac{(f_H - f_L)N_1}{\pi B_f^2} \quad (34)$$

Equations (32), (33), and (34) characterize the constant sweep rate spectrum analyzer for periodic signals.

The restrictions imposed by equation (3) have not yet been incorporated into the design equations. Assume that the filter center frequency f_c in a swept frequency analyzer with modulation is greater than 10 kilohertz. Then, from equation (3), the detector averaging time must be much greater than 100 microseconds. This criterion is easily met and in practical cases imposes no further restrictions on the sweep rate as given by equation (32).

The design of a constant sweep rate spectrum analyzer for periodic signals is straightforward. A filter bandwidth B_f is chosen to give what the user considers to be adequate resolution, with equation (1) used as a guide. The sweep rate is given by equation (32), the frequency-time function to be implemented by a variable frequency oscillator is given by equation (33), and the total sweep time is given by equation (34).

Notice in equation (32) that the sweep rate is directly proportional to B_f^2 , while the total sweep time in equation (34) is inversely proportional to B_f^2 . As a numerical example consider the following:

$$B_f = 10 \text{ Hz}$$

$$f_H = 2000 \text{ Hz}$$

$$f_L = 100 \text{ Hz}$$

Choose

$$N_1 = \pi$$

Then,

$$K_1 = 100 \text{ Hz/sec}$$

$$f_i(t) = 100t + 100$$

$$t_s = 19 \text{ sec}$$

This example has good resolution but a fairly long sweep time. If B_f were expanded to 20 hertz, the total sweep time would be only 4.8 seconds. The resolution would not have been degraded significantly; however, the system still would not meet the desired specifications listed in the INTRODUCTION because the minimum resolvable bandwidth would have been increased to 20 percent of the bandwidth center frequency at f_L .

Note that this analysis only applies to periodic data where there is no statistical uncertainty in the filter output. Spectrum analyzers for random data are discussed in the next section.

Design for Random Signals

Basic sweep rate equation. - Consider analyzing a spectral peak of width B_n with a filter of width B_f . First, to limit the amplitude error to ϵ with a confidence level $1-\alpha$, the filter must be entirely within the peak B_n for a time T determined from

$$\epsilon = \eta \sqrt{\frac{2}{2B_f T - 1}} \quad (28)$$

The use of this equation assumes that $2B_f T - 1$ is sufficiently large that the normal approximation to the chi-square distribution may be used.

Secondly, it can be shown that continuously sweeping a filter through a spectrum yields approximately the same results as moving the filter through it in discrete steps (ref. 7). From these considerations the filter can only be swept $B_n - B_f$ in frequency in a time T and still remain entirely within the peak for the duration T . Thus,

$$\frac{df_i}{dt} \approx \frac{\Delta f_i}{\Delta t} = \frac{B_n - B_f}{T} \quad (35)$$

This is the basic sweep rate equation.

Optimization of the filter bandwidth. - For in-flight vibration data the mechanical Q was assumed to be independent of the resonant frequency. Thus, if the minimum resolvable peak with center frequency f_i is defined as B_n , then the following equation results from the universal resonance curve:

$$B_n = \frac{f_i}{Q_{\max}} \quad (36)$$

Clearly, if a filter of bandwidth B_f is to resolve a spectral peak of width B_n , then B_f must be less than B_n . Let

$$B_f = C B_n \quad (37)$$

where C is a constant to be determined. For a fixed bandwidth filter

$$B_f = C(B_n)_{\min} = \frac{C f_L}{Q_{\max}} \quad (38)$$

since the filter must be able to resolve a spectral peak at the frequency f_L . Consider the optimum value for B_f or C . This means given ϵ and η find B_f such that df_i/dt is a maximum; that is, solve for

$$\frac{d}{dB_f} \left(\frac{df_i}{dt} \right) = 0$$

and

$$\frac{d^2}{dB_f^2} \left(\frac{df_i}{dt} \right) < 0$$

Solving equation (28) for T and substituting in the basic sweep rate equation yield

$$\frac{df_i}{dt} = \frac{(B_n - B_f) 2 B_f \epsilon^2}{2\eta^2 + \epsilon^2} \quad (39)$$

Now

$$\frac{d}{dB_f} \left(\frac{df_i}{dt} \right) = \frac{(B_n - 2B_f) 2\epsilon^2}{2\eta^2 + \epsilon^2} = 0$$

Thus,

$$B_f = \frac{B_n}{2} \text{ or } C = \frac{1}{2}$$

Since

$$\frac{d^2}{dB_f^2} \left(\frac{df_i}{dt} \right) = - \frac{4\epsilon^2}{2\eta^2 + \epsilon^2} < 0$$

the sweep rate is truly a maximum. Note that C is a constant independent of ϵ and η .

This optimization of B_f/B_n ignores blurring error.¹ However, consideration of the blurring error shows the apparent increase in bandwidth to be comparable to the resolution and the frequency accuracy as given in the list of desired specifications (p. 3). For $B_f/B_n = 1/2$ (the optimum ratio shown above), Ratz (ref. 6) showed that if a fairly sharp filter is used, the apparent increase in bandwidth is about 10 percent.

Application

Linear sweep (analyzer I). - Using the basic sweep equation

$$\frac{df_i}{dt} = \frac{B_n - B_f}{T} \quad (35)$$

the desired types of sweeps can be generated. Assume B_n , B_f , and T fixed. Thus,

$$\frac{df_i}{dt} = \frac{B_n - \frac{1}{2} B_n}{T} = \frac{f_L}{2Q_{\max} T}$$

or

$$f_i = \frac{f_L t}{2Q_{\max} T} + f_L \quad (40)$$

¹The blurring error is defined by Ratz (ref. 6) as an apparent increase in the bandwidth of the spectral peaks, the increase being due to the nonzero bandwidth of the filter and the shape of the transfer function.

This equation is for a linear sweep spectrum analyzer, which for brevity is called analyzer I in this report. Such a system has constant frequency resolution rather than constant-percent frequency resolution; thus, when analyzing vibration data, much time is wasted scanning the high-frequency end of the spectrum.

Nonlinear sweeps (analyzers II and III). - If $B_n = f_i/Q_{\max}$ as previously stated and B_f remains fixed, then the sweep rate equation becomes

$$\frac{df_i}{dt} = \frac{\frac{f_i}{Q_{\max}} - B_f}{T}$$

or

$$f_i = B_f Q_{\max} + (f_L - B_f Q_{\max}) e^{\frac{t}{T Q_{\max}}} \quad (41)$$

In this case the frequency is an exponential function of time. Thus the system is normally called an exponential sweep spectrum analyzer; however, it is called analyzer II in this report.

As stated previously, this system is optimum for a fixed bandwidth filter and $B_n = f_i/Q_{\max}$; however, the filter bandwidth is optimum only at $f_i = f_L$, where $B_f = (B_n)_{\min}/2$. Thus at the high end of the spectrum time is wasted because the filter is too narrow. This leads to the following considerations. Assume the existence of a variable bandwidth filter (ref. 14). Then the optimum filter bandwidth is

$$B_f = \frac{B_n}{2} = \frac{f_i}{2Q_{\max}}$$

Again the sweep rate equation is

$$\frac{df_i}{dt} = \frac{B_n - B_f}{T}$$

However, B_f and T are no longer constants; hence, B_f and T must be substituted into the sweep rate equation in terms of f_i . Using equation (28) and $B_f = f_i/2Q_{\max}$ yields

$$\frac{df_i}{dt} = \frac{\epsilon^2 f_i^2}{Q_{\max}^2 (4\eta^2 + 2\epsilon^2)}$$

or

$$f_i = \frac{f_L}{1 - \frac{f_L^2 \epsilon^2 t}{Q_{\max}^2 (4\eta^2 + 2\epsilon^2)}} \quad (42)$$

This system will be subsequently designated analyzer III. Note that the sweep rate is proportional to f_i^2 . For a comparison of the above systems consider the following example with

$$B_f = 5 \text{ Hz}$$

$$f_L = 100 \text{ Hz}$$

$$f_H = 2000 \text{ Hz}$$

$$\epsilon = 10 \text{ percent}$$

$$1 - \alpha = 90 \text{ percent}$$

$$Q_{\max} = 10$$

From the normal distribution curve and the preceding value for $1 - \alpha$, $\eta = 1.64$ in equation (28); then substituting $\epsilon = 10$ percent into equation (28) yields $T = 54.2$ seconds. Solving equations (40), (41), and (42) for the sweep time in all three systems yields the following equations:

Analyzer I:

$$t_s = \frac{(f_H - f_L)T}{B_f} = \frac{(f_H - f_L)(2\eta^2 + \epsilon^2)}{2B_f^2 \epsilon^2} \quad (43)$$

Analyzer II:

$$t_s = Q_{\max} T \ln \frac{f_H - B_f Q_{\max}}{f_L - B_f Q_{\max}}$$

$$= \frac{Q_{\max} (2\eta^2 + \epsilon^2)}{2\epsilon^2 B_f} \ln \frac{f_H - B_f Q_{\max}}{f_L - B_f Q_{\max}} \quad (44)$$

Analyzer III:

$$t_s = \frac{2(f_H - f_L)Q_{\max}^2 (2\eta^2 + \epsilon^2)}{f_H f_L \epsilon^2} \quad (45)$$

Substituting the appropriate values of f_H , f_L , T , B_f , Q_{\max} , η , and ϵ into equations (43), (44), and (45) yields

$$t_s = 20\,600 \text{ sec} \quad \text{for analyzer I}$$

$$t_s = 1980 \text{ sec} \quad \text{for analyzer II}$$

$$t_s = 1030 \text{ sec} \quad \text{for analyzer III}$$

The nonlinear sweep systems have much shorter sweep times than the linear system; however, none are feasible for real-time analysis of in-flight vibration data with the accuracy and sweep time requirements given in the INTRODUCTION. Clearly, if t_s is to be on the order of seconds, then accuracy, resolution, and confidence must be sacrificed. The following example was chosen to show to what extent the requirements must be degraded to obtain sweep times on the order of seconds. Assume

$$\eta = 0.84 \text{ (i. e.) } 1 - \alpha = 60 \text{ percent}$$

$$f_L = 100 \text{ Hz}$$

$$f_H = 2000 \text{ Hz}$$

$$Q_{\max} = 5$$

$$\epsilon = 40 \text{ percent}$$

For analyzer I

$$t_s = 91.6 \text{ sec}$$

For analyzer II

$$t_s = 8.9 \text{ sec}$$

For analyzer III

$$t_s = 4.6 \text{ sec}$$

In the first numerical example, the value for the degrees of freedom is 540; so there is no doubt about the normal distribution being a good approximation to the chi-square distribution. In the second example there are 9 degrees of freedom. The normal approximation gives

$$P \left[0.60 \leq \frac{\hat{S}(f_c)}{S(f_c)} \leq 1.40 \right] = 0.60$$

The chi-square distribution gives

$$P \left[0.598 \leq \frac{\hat{S}(f_c)}{S(f_c)} \leq 1.36 \right] = 0.60$$

In view of the fact that ϵ and α have been chosen to be 40 percent, the agreement between the normal approximation and the chi-square distribution is more than adequate.

BANDWIDTH REQUIRED FOR TRANSMISSION OF SPECTRUMS

Single Spectrum Bandwidth Requirement

Transmitting the spectrum of a signal rather than transmitting the time function will lead to a considerable saving in bandwidth. Consider an airborne spectrum analyzer with a resistor-capacitor (RC) averager on the power detector. Then, according to the discussion following equation (13), $RC = T/2$. But according to equation (28)

$$T = \frac{2\eta^2 + \epsilon^2}{2B_f\epsilon^2} \quad (46)$$

For good response, the transmission channel should have a time constant shorter than the time constant of the data entering the channel; that is,

$$\frac{\text{data time constant}}{\text{channel time constant}} \equiv r > 1 \quad (47)$$

The time constant of the channel response is $1/\pi B_c$ from equation (2), where B_c is the transmitting channel bandwidth. Substituting these results into equation (47) yields

$$\frac{RC}{1/\pi B_c} = r$$

or

$$B_c = \frac{4rB_f\epsilon^2}{\pi(2\eta^2 + \epsilon^2)} \quad (48)$$

This expression is valid for all three types of sweep functions. For analyzers I and II, B_f is constant and equal to $f_L/2Q_{\max}$, while for analyzer III the maximum value of B_f is used in equation (48); that is, $B_{f, \max} = f_H/2Q_{\max}$. As a result, analyzer III requires substantially more bandwidth than analyzers I and II. The reason for the different bandwidth requirements is best understood by considering the power detector averaging time. In analyzers I and II, the averaging time of the power detector is constant, while in analyzer III the averaging time is inversely proportional to frequency. Thus, at the higher frequencies the power detector in analyzer III responds much faster than the detector in the other systems. Providing the bandwidth to transmit the more rapid changes in power level from analyzer III accounts for the increased bandwidth. The channel bandwidth B_c is calculated for all three types of analyzers using the values of B_f , ϵ , and η of the previous sweep time examples. The values for B_c are given in table I for the following conditions:

TABLE I. - NUMERICAL EXAMPLES

Analyzer	Total sweep time, t_s , sec	Transmitting channel bandwidth, B_c , Hz	Telemetry bandwidth per channel, $\frac{B_M}{m}$, Hz/channel	Telemetry bandwidth per channel for PCM system, $\left(\frac{B_M}{m}\right)_{\text{PCM}}$, Hz/channel
Example 1				
I	20 600	0.037	0.082	0.22
II	1 980	.037	.082	.22
III	1 030	.74	.164	4.4
Example 2				
I	91.6	4.1	9.1	25
II	8.9	4.1	9.1	25
III	4.6	82	182	500

Example 1

Example 2

Lowest frequency analyzed, f_L , Hz	100	100
Highest frequency analyzed, f_H , Hz	2000	2000
Percent error, ϵ	10	40
Confidence level, $1-\alpha$, percent	90	60
Maximum mechanical Q, Q_{\max}	10	5
Standardized random variable, η	1.645	0.84
Ratio of data time constant to channel time constant, r	π	π

Example 1 in the table is impractical because of the long sweep times. In example 2, B_c for all the analyzers shows that spectrum transmission results in a substantial bandwidth saving over time function transmission; that is, 1900 hertz of data has been reduced to less than 100 hertz.

Multispectrum Bandwidth Requirements

Consider the transmission of the spectrums from many channels of wide-band data. Frequency multiplexing or time division multiplexing might be used to combine the spectrums. Since the transmitting channel bandwidths, as calculated in the numerical ex-

amples, are so small, frequency stacking of the spectrum channels would be inefficient. More bandwidth would be used for channel separation than for information, and narrow, complex filters would be required. Time division multiplexing is more easily implemented.

Consider the following sampling method. Assume each spectrum analyzer output to be band limited to frequencies less than B_c . Then from sampling theory, (ref. 12), the theoretical minimum sampling rate is $2B_c$. Thus, to multiplex m spectrum channels, $2mB_c$ samples per second are required.² The amplitudes of the $2mB_c$ samples per second for m spectrum channels can be transmitted with an accuracy of about 3 percent; that is, $e^{-3.5}$ by using a telemetry channel whose bandwidth B_M is calculated from

$$3.5 \text{ (channel time constant)} = \frac{3.5}{\pi B_M} = \frac{1}{2mB_c} \quad (49)$$

or

$$B_M = \frac{7mB_c}{\pi} \quad (50)$$

The quantity B_M/m , the telemetry bandwidth per spectrum channel, may be used as a figure of merit for comparisons to other data transmission systems:

$$\frac{B_M}{m} = \frac{7B_c}{\pi} = \frac{28}{\pi^2} \frac{r B_f \epsilon^2}{2\eta^2 + \epsilon^2} \quad (51)$$

Values of B_M/m were calculated for the two examples, and the results are given in table I. These results show that even after multiplexing, a large bandwidth saving still remains.

²The use of the theoretical minimum sampling rate based on the frequency limit B_c is sufficient in a practical system because of the frequency-limiting effects of the RC averager used on the power detector of the spectrum analyzer. The cut-off frequency of the RC averager is B_c/π . Thus, the frequency components in the analog signal representing the spectrum are attenuated at frequencies above B_c/π . The lower frequency components of the signal, which comprise the major part of the signal, are sampled many times per cycle, which in a practical system yields good reproduction of the sampled spectrum.

If each sample were to be converted into a five-bit digital word, which would give an accuracy of about 3 percent, and an extra bit were added for synchronization, the bit rate for coding m spectrum channels would be $12mB_c$ bits per second. It is a well-known rule in binary pulse transmission that through a low-pass channel that transmits up to a frequency of f_d hertz, one can send $2f_d$ pulses per second. Thus, for this binary-coded sampling scheme (pulse code modulation) the multiplexed bandwidth per spectrum channel $\left(\frac{B_M}{m}\right)_{PCM}$ is given by

$$\left(\frac{B_M}{m}\right)_{PCM} = 6B_c = \frac{24r B_f \epsilon^2}{\pi(2\eta^2 + \epsilon^2)} \quad (52)$$

Values of $\left(\frac{B_M}{m}\right)_{PCM}$ are also contained in table I. The bandwidth equations (48), (51), and (52) take into account only the bandwidth required for the transmission of the spectrums. Any low frequencies not included in the spectrum analysis must be transmitted separately.

On the basis of these bandwidth calculations, analyzer II appears to be better suited for airborne use. In the two examples, analyzer II has a large bandwidth advantage (a factor of 20) and has a total sweep time less than twice that of analyzer III. Analyzer I has limited use because of its extremely long sweep times.

CONCLUSIONS

The optimum frequency-time functions were developed for three types of swept frequency spectrum analyzers. The functions were optimized by tailoring the design to the characteristics of the expected signals. The analysis for the linear and the exponential sweep analyzers (analyzers I and II, respectively) can be applied to systems that have already been developed. The frequency-time equation for the second-order sweep-rate spectrum analyzer (analyzer III) can be used as the basic design equation for the development of an improved spectrum analyzer having a short sweep time.

The performance of in-flight spectrum analyzers on random data was calculated to be far short of the performance specifications desired. In order to obtain sweep times of the order of seconds, decreased resolution and/or reduced accuracy are required.

The exponential sweep spectrum analyzer was judged to be the best spectrum analyzer for in-flight use. Compared with the analyzer with the second-order sweep rate, the exponential sweep analyzer has only moderately longer sweep times (about two to one), requires less bandwidth (a ratio of 20 to 1), and is less complex. Both of the spectrum

analyzers with time-dependent sweep rates have sweep times about 10 times shorter than the constant sweep rate analyzer.

Finally, the bandwidth per channel was calculated for the transmission of spectrums. Spectrum analysis showed substantial bandwidth savings over the transmission of wide-band data (at least 75 to 1). Many channels of spectrums can be transmitted over one wide-band channel of an IRIG FM/FM system.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 20, 1966,
125-24-03-03-22.

APPENDIX - SYMBOLS

B_c	transmitting channel bandwidth for one channel of continuous spectrum information
B_f	effective filter bandwidth (bandwidth of the ideal rectangular filter that passes the same power as the real filter when both are excited by white noise)
B_M	telemetry bandwidth for m channels of sampled spectrums
$\left(\frac{B_M}{m}\right)_{PCM}$	multiplexed bandwidth per spectrum channel for PCM system
B_n	minimum resolvable spectrum peak with center frequency f_i
C	ratio of B_f to $B_{n, \min}$
$E[y]$	statistical expected value of y
$E[y^2]$	statistical second moment of y
f_a, f_b	lower and upper frequency limits on effective bandwidth of analyzing filter
f_c	analyzing filter center frequency
f_H	highest frequency to be analyzed
f_i	signal frequency being analyzed
$\frac{df_i}{dt}$	sweep rate
f_L	lowest frequency to be analyzed
f_s	mixing frequency
K_1	sweep rate constant
m	number of channels
N	number of samples taken to determine sample variance
N_1	numerical constant that allows adequate filter response for periodic signals
n	number of independent samples obtainable from band-limited white noise

$P[a \leq y \leq b]$	probability of $a \leq y \leq b$
P_{ab}	average power in $x(t)$ contained between frequencies f_a and f_b
\overline{P}_{ab}	average power out of filter of bandwidth B_f , spanning f_a to f_b
$p[x(t)]$	probability distribution of $x(t)$
Q	mechanical Q
$R(\tau)$	autocorrelation function
RC	time constant of resistor-capacitor averaging circuit
r	ratio of data time constant to channel time constant
$S(f)$	power spectral density
$S(f_c)$	exact value of power spectral density at f_c
$\hat{S}(f_c)$	estimated value of power spectral density at f_c
$\hat{S}_{ab}(f)$	estimated value of average power spectral density between frequencies f_a and f_b
S_1, S_2	lower and upper limits on confidence interval
s^2	sample variance
T	limit on integrations with respect to time; also used as averaging time for power detector
T_c	time constant of filter response to sine wave
T_d	detector averaging time
T_p	signal period for periodic signals
t	time
t_s	total sweep time
$\text{Var } x(t)$	variance of $x(t)$
w	normal random variable with mean μ and standard deviation σ
X_1, X_2	specific values of chi-square random variable; the values are those which give the desired confidence level
$x(t)$	signal (random or periodic)

$x_f(t)$	filter output
z	standardized normal random variable with zero mean and a standard deviation of one
α	fraction of measured values of the power spectral density that will lie outside the confidence interval
ϵ	percent error
η	standardized random variable that gives desired confidence level
μ	mean value
σ^2	true variance
τ	lead time in autocorrelation computation
χ^2_{N-1}	chi-square random variable with $N-1$ degrees of freedom

Subscripts:

max	maximum
min	minimum

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